← laplace transform or laplace transformation →

$$L\{F(t)\} = \int_0^\infty e^{-pt} F(t)\,dt = F(p)$$

Note this integral should exist and p is a parameter may be real or complex.

Some notations → (generally used)

$$L\{F(t)\} = \bar{F}(p)$$

or

$$L\{y(t)\} = \bar{y}(p)$$

or $L\{F(t)\} = \phi(p)$ etc.

use of laplace transform →

- used by scientists and engineers in solving linear differential eq^n – ordinary as well as partial
- it reduces an ordinary differential eq^n into an algebraic eq^n.

properties of laplace transformation →

$$L\{c_1F_1(t) + c_2F_2(t)\} = c_1L\{F_1(t)\} + c_2L\{F_2(t)\}.$$

{c_1 & c_2 are const.

1. laplace transform of $F(t) = 1$

$$L\{f(t)\} = L\{1\} = \int_0^\infty e^{-pt}.1.dt = \frac{1}{p}$$

$\boxed{L\{1\} = \frac{1}{p}}$, $p > 0$

2. laplace transform of the fun^n $F(t) = t^n$, n is any real number.

$$L\{f(t)\} = L\{t^n\} = \int_0^\infty f(t).e^{-pt}.dt$$

$$= \int_0^\infty t^n . e^{-pt} .dt = \int_0^\infty e^{-pt}\, t^{(n+1)-1}\, dt$$

$$\left\{ \because \int_0^\infty e^{-at} . t^{n-1} dt = \frac{\Gamma n}{a^n}, \beta\text{-fun}^n \right.$$

$$= \frac{\Gamma(n+1)}{p^{n+1}}$$

$$\Rightarrow \boxed{L\{t^n\} = \frac{\Gamma(n+1)}{p^{n+1}}}^{*}, \quad p > 0, \text{ n is any real no.}$$

$$\boxed{L\{t^n\} = \frac{n!}{p^{n+1}}} \leftarrow \text{if n is +ve integer.}$$

$$\boxed{L\{t\} = \frac{1}{p^2}}$$

3. $$\boxed{L\{e^{at}\} = \frac{1}{p-a}}, \quad p > a$$

4. $$\boxed{L\{\sin at\} = \frac{a}{a^2+p^2}}, \quad p > 0$$

5. $\boxed{L\{\cos at\} = \frac{p}{a^2+p^2}}$, $p>0$

6. $\boxed{L\{\sinh at\} = \frac{a}{p^2-a^2}}$, $p>|a|$

or

$L\left\{\frac{e^{at}-e^{-at}}{2}\right\}$

to retain in mind.

$\sin^2\theta + \cos^2\theta = 1$

but $\cosh^2\theta - \sinh^2\theta = 1$

$= \frac{1}{2}L\{e^{at}\} - \frac{1}{2}L\{e^{-at}\}$

by linear property

$\sinh at = \frac{e^{at}-e^{-at}}{2}$

$= \frac{1}{2}\cdot\frac{1}{p-a} - \frac{1}{2}\cdot\frac{1}{p+a}$

$\cosh at = \frac{e^{at}+e^{-at}}{2}$

$\Rightarrow \frac{1}{2}\left[\frac{(p+a)-(p-a)}{(p-a)(p+a)}\right]$

$L\{e^{at}\} = \frac{1}{p-a}$

$L\{e^{-at}\} = \frac{1}{p+a}$

$= \frac{1}{\cancel{2}}\left[\frac{\cancel{2}a}{p^2-a^2}\right]$

$\Rightarrow \boxed{L\{\sinh at\} = \frac{a}{p^2-a^2}}$

7. $\boxed{L\{\cosh at\} = \frac{p}{p^2-a^2}}$, $p>|a|$

S.No.	$f(t)$	$L\{f(t)\}$
1.	1	$\frac{1}{p}, \; p > 0$
2.	t	$\frac{1}{p^2}, \; p > 0$
3.	t^n (n is real & $n > -1$)	$\frac{\Gamma(n+1)}{p^{n+1}}$ or $\frac{n!}{p^{n+1}}, \; p > 0$
4.	$L\{e^{at}\}$	$\frac{1}{p-a}, \; p > a$
5.	$L\{e^{-at}\}$	$\frac{1}{p+a},$
6.	$L\{\sin at\}$.	$\frac{a}{a^2+p^2}, \; p > 0$
7.	$L\{\cos at\}$	$\frac{p}{p^2+a^2}, \; p > 0$
8.	$L\{\sinh at\}$	$\frac{a}{p^2-a^2}, \; p > \lvert a \rvert$
9.	$L\{\cosh at\}$	$\frac{p}{p^2-a^2}, \; p > \lvert a \rvert$
10.		

$\Gamma(1/2) = \sqrt{\pi}$

$\Gamma(3/2) = \frac{1}{2}\sqrt{\pi}$

$\Gamma(5/2) = \frac{3}{2} \cdot \frac{1}{2}\sqrt{\pi}$

$\cosh^2 t + \sinh^2 t = \cosh(2t) = 1 + 2\sinh^2 t$
$= 2\cosh^2 t - 1$

$\sinh 3t = 3\sinh t + 4\sinh^3 t$

$\cosh 3t = 4\cosh^3 t - 3\cosh t$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots$$

Existence of laplace transform →

if $F(t)$ is a function which is piecewise Continuous on every finite interval in the range $t \geq 0$, and satisfies

$$|f(t)| \leq Me^{at} \quad \forall \; t \geq 0$$

& for some constants a and M, then the laplace transform of $F(t)$ exists $\forall \; p > a$.

properties of laplace transform →

1. shifting / translation property →

if $L\{f(t)\} = F(p)$ then.

$$L\{e^{at} f(t)\} = F(p-a) \quad \& \quad L\{e^{-at} f(t)\} = F(p+a).$$

2. change of scale property →

if $L\{F(t)\} = F(p)$, then $L\{f(at)\} = \frac{1}{a} F\left(\frac{p}{a}\right)$.

3. Heaviside shifting theorem / second shifting property →

if $L\{F(t)\} = F(p)$ &
$$G(t) = \begin{cases} f(t-a), & t > a \\ 0 & t < a \end{cases}$$

then.
$$L\{G(t)\} = e^{-ap} F(p)$$

3. Multiplication by t^n. →

if $L\{f(t)\} = F(p)$ then

where

$$L\{t^n f(t)\} = (-1)^n \frac{d^n}{dp^n} F(p), \quad n = 1, 2, 3, \ldots$$

if $n=1$.

$$L\{t f(t)\} = -F'(p)$$

if $n=2$

$$L\{t^2 f(t)\} = \frac{d^2}{dp^2} F(p) \text{ or } = F''(p).$$

4. Division by t →

if $L\{F(t)\} = F(p)$ then

$$L\left\{\frac{F(t)}{t}\right\} = \int_p^\infty F(p) \cdot dp$$ provided the integral exist.

5. Laplace transform of derivatives →

if $L\{F(t)\} = F(p)$, then

- $L\{f'(t)\} = pF(p) - f(0) = pL\{f(t)\} - f(0)$
- $L\{f''(t)\} = p^2 F(p) - pf(0) - f'(0)$
- $L\{f'''(t)\} = p^3 F(p) - p^2 f(0) - pf'(0) - f''(0)$

6. Laplace transform of integral →

if $L\{f(t)\} = F(p)$ then.

$$L\left\{\int_0^t f(t)\cdot dt\right\} = \frac{F(p)}{p}.$$

7. Laplace transform of periodic function →

if $f(t)$ is a periodic function of period T

then
$$L\{f(t)\} = \frac{1}{1-e^{-pT}}\int_0^T e^{-pt}\cdot f(t)\cdot dt$$

← unit step function or Heaviside unit step function →

The unit step fun" $u(t-a)$ is defined as: →

$$u(t-a) = \begin{cases} 0, & t<a \\ 1 & t\geq a \end{cases}$$

Note

at $a=0$ then.

$$u(t) = \begin{cases} 0, & t<0 \\ 1 & t\geq 0 \end{cases}$$

Laplace transform of unit step function →

$$L\{u(t-a)\} = \int_0^\infty e^{-pt}\, u(t-a)\cdot dt$$

$$= \cancel{\int_0^a e^{-pt}\cdot(0)\cdot dt} + \int_a^\infty e^{-pt}\cdot 1\cdot dt$$

$$\boxed{L\{u(t-a)\} = \frac{e^{-ap}}{p},} \quad p>0$$

if $a=0$

then. $L\{u(t)\} = \frac{1}{p}$.

unit step func" with second shifting theorem→

We have
$$u(t-a) = \begin{cases} 0, & t<a \\ 1, & t \geq a \end{cases}$$

$$F(t-a)u(t-a) = \begin{cases} 0, & t<a \\ f(t-a), & t \geq a \end{cases} = G(t) \text{ say.}$$

Then by second shifting theorem→

$$L\{G(t)\} = e^{-ap} F(p)$$

$$L\{f(t-a)u(t-a)\} = e^{-ap} F(P) = e^{-ap} L\{f(t)\}$$

if $a=0$.

then. $L\{f(t)\, u(t)\} = F(p)$.

Inverse laplace transform → please note : inverse of LT may not be unique

if $L\{f(t)\} = F(p)$ then. $L^{-1}\{F(p)\} = f(t)$.

Laplace transform	Inverse laplace transform
1. $L\{1\} = \frac{1}{p}$	$L^{-1}\{\frac{1}{p}\} = 1$
2. $L\{e^{at}\} = \frac{1}{p-a}$	$L^{-1}\{\frac{1}{p-a}\} = e^{at}$
3. $L\{t^n\} = \frac{n!}{p^{n+1}}$	$L^{-1}\{\frac{1}{p^{n+1}}\} = \frac{t^n}{n!}$

laplace transform

$L\{\sin at\} = \frac{a}{a^2+p^2}$

$L\{\cos at\} = \frac{p}{p^2+a^2}$

$L\{\sinh at\} = \frac{a}{p^2-a^2}$

$L\{\cosh at\} = \frac{p}{p^2-a^2}$

$L\{c_1 f_1(t)\} + \cancel{L}\{c_2 f_2(t)\}$

$= c_1 L\{f_1(t)\} + c_2 L_2\{f_2(t)\}$

Inverse laplace transform

$L^{-1}\left\{\frac{a}{p^2+a^2}\right\} = \sin at$

$L^{-1}\left\{\frac{p}{p^2+a^2}\right\} = \cos at$

$L^{-1}\left\{\frac{a}{p^2-a^2}\right\} = \sinh at$

$L^{-1}\left\{\frac{p}{p^2-a^2}\right\} = \cosh at$

$L^{-1}\{c_1 F_1(p) + c_2 F_2(p)\}$

$= c_1 L^{-1}\{F_1(p)\}$

$+ c_2 L^{-1}\{F_2(p)\}$

* $(1+z)^{-1} = 1 - z + z^2 - z^3 + \cdots$

1. first ~~Inverse~~ shifting or translation theorem →

if $L^{-1}\{F(p)\} = f(t)$ then

$L^{-1}\{F(p-a)\} = e^{at} f(t). = e^{at} L^{-1}\{F(p)\}.$

2. Second shifting or translation property →

if $L^{-1}\{F(p)\} = f(t)$ then $L^{-1}\{e^{-ap} F(p)\} = G(t)$

where $G(t) = \begin{cases} f(t-a), & t \geq a \\ 0, & t < a \end{cases}$

second shifting property in term of unit step fun:

if $L^{-1}\{F(p)\} = f(t)$ then $L^{-1}\{e^{-ap}F(p)\} = f(t-a)u(t-a)$.

3. Change of scale property →

if $L^{-1}\{F(p)\} = f(t)$ then.

$$L^{-1}\{F(ap)\} = \frac{1}{a} f\left(\frac{t}{a}\right)$$

tan(x)=x+(x^3)/3+(2x^5)/15+(17x^7)/315+(62x^9)/2835

coshx = 1+x^2/2!+ x^4/4!   (alternate +- unlike cosx && simillar with sinh case)

tan^-1x =x-x^3/3 +x^5/5   (similar with sine fun without !)

e^x = 1+x +x^2/2!+ x^3/3!.....................